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NATIONAL LEVEL SCIENCE TALENT SEARCH EXAMINATION (UPDATED)

CLASS - 11 (PCM)
Question Paper Code : UN499

KEY

1. D	2. A	3. D	4. A	5. B	6. C	7. B	8. B	9. D	10. B
11. C	12. A	13. A	14. C	15. B	16. A	17. C	18. A	19. B	20. C
21. D	22. D	23. C	24. B	25. C	26. B	27. B	28. D	29. A	30. D
31. A	32. C	33. D	34. B	35. D	36. D	37. B	38. D	39. C	40. A
41. A	42. B	43. D	44. C	45. C	46. A	47. D	48. D	49. B	50. B
51. B	52. B	53. B	54. C	55. A	56. D	57. A	58. D	59. B	60. C

EXPLANATIONS

MATHEMATICS

1: (D) $\left(\frac{8}{5}\right)^{1-x^2} > \left(\frac{5}{8}\right)^{6(1+x)}$

$1 - x^2 > -6(1 + x)$

$\Rightarrow x^2 - 6x - 7 < 0 \Rightarrow x \in (-1, 7)$

02. (A) (fofof) (–1) + (fofof) (0) + (fofof) (1)

$= -2 + 33 - 2 = 29$; $f(4\sqrt{2}) = 32 - 3 = 29$

03. (D) $xyz = (p + q)(p\omega + q\omega^2)(p\omega^2 + q\omega)$

$= p^3 + q^3$

04. (A) $\frac{y}{1} = \frac{x^2 - x + 1}{x^2 + x + 1}$

$x^2 - x + 1 = x^2y + xy + y$

$(1 - y)x^2 + (-1 - y)x + (1 - y) = 0$

$(1 + y)^2 - 4(1 - y)^2 > 0$

$1 + y^2 + 2y - 4(1 + y^2 - 2y) > 0$

$1 + y^2 + 2y - 4 - 4y^2 + 8y \geq 0$

$-3y^2 + 10y - 3 \geq 0$

$3y^2 - 10y + 3 < 0$

$3y^2 - 9y - y + 3 < 0$

$3y(y - 3) - 1(y - 3) \leq 0$

$y \in \left[\frac{1}{3}, 3\right]$

$\therefore$ Minimum value = $\frac{1}{3}$

05. (B) $30c_2 - 8c_2 + 1$

$= \frac{30\times29}{2} - \frac{8\times7}{2} + 1$

= 15 × 29 – 28 + 1

= 435 – 28 +1

= 436 – 28

= 408

06. (C) $\frac{1}{a^3}\left[1+\frac{b}{a}x\right]^{-3} = \frac{1}{27}+\frac{x}{3}+.......$

$\Rightarrow \frac{1}{a^3}\left[1-\frac{3b}{a}x+......\right] = \frac{1}{27}+\frac{x}{3}$

$\Rightarrow \frac{1}{a^3} = \frac{1}{27} = a = 3$

$-\frac{3b}{a^4}x = \frac{x}{3} = -\frac{3b}{27} = \frac{1}{3} \quad b = -9$

$\therefore (3,-9)$

07. (B) $\frac{27\tan^2\theta+3\cot^2\theta}{2} \geq \sqrt{27\tan^2\theta\times3\cot^2\theta}$

[$\because$ AM $\geq$ GM]

$\therefore$ 27 $\tan^2\theta$ + 3$\cot^2\theta \geq 2 \times 9$

$\therefore$ 27 $\tan^2\theta$ + 3$\cot^2\theta \geq 18$

$\therefore$ Minimum value of 27 $\tan^2\theta$ + 3$\cot^2\theta$ = 18

08. (B) cos 36° – cos 72°

$= \frac{\sqrt{5}+1}{4} - \frac{\sqrt{5}-1}{4} = \frac{1}{2}$

09. (D) (k, 2k),(3k, 3k),(3, 1) are collinear

$\Rightarrow K = \frac{-1}{3}$

Education of the line l is $y-1=\frac{1}{2}(x-3)$

$\Rightarrow -x - 2y - 1 = 0$

Distance from origin is $\frac{1}{\sqrt{1+4}} = \frac{1}{\sqrt{5}}$

10. (B) Centroid of ΔABC = Centroid of ΔDEF

$\therefore G\left(\frac{4}{3},\frac{2}{3},0\right)$

11. (C) Focus (a, 0) = (3, 0) a = 3

Directrix x + a = 0 $\Rightarrow x + 3 = 0$

$\therefore$ Equation of parabola is $y^2 = 4ax = 12x$

12. (A) The y-coordinate of foci is zero

$\therefore$ Major axis is on X-axis ae = 4

Let, equation of ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

[$\because b^2 = a^2(1 - e^2) = a^2 - 16$]

$\Rightarrow \frac{32}{a^2}+\frac{24}{a^2-16}=1$

$\Rightarrow 32a^2 - 512 + 24a^2 = a^2(a^2 - 16)$

$\Rightarrow 56a^2 - 512 = a^4 - 16a^2$

$\Rightarrow a^4 - 72a^2 + 512 = 0$

$\Rightarrow a^2 - 64a^2 - 8a^2 + 512 = 0$

$\Rightarrow a^2(a^2 - 64) - 8(a^2 - 64) = 0$

$\Rightarrow (a^2 - 8)(a^2 - 64) = 0$

$\Rightarrow a^2 = 64 \Rightarrow a = 8$ ($\because a^2 = 8$ is not possible)

$\because$ ae = 4 $\Rightarrow$ 8 × e = 4

$\Rightarrow$ e = $\frac{1}{2}$

13. (A) The number of persons in a selection may be of the following types

(i) Man's relative (0 males + 3 Females) + wife's relatives (3 Males + 0 Females)

(ii) Man's relatives (1 male + 2 Females) + wife's relatives (2 males + 1 Female)

(iii) Man's relatives (2 males + 1 Female) + wife's relatives (1 male + 2 Females)

(iv) Man's relatives (3 males + 0 Females) + wife's relatives (0 males + 3 Females)

$\therefore$ The required number of selections

$= {}^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0 \times {}^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1 \times {}^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2 \times {}^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3$

$= 1 \times 4 \times 4 \times 1 + 3 \times 6 \times 6 \times 3 + 3 \times 4 \times 4 \times 3 + 1 \times 1 \times 1 \times 1$

$= 16 + 324 + 144 + 1 = 485$

14. (C) $\underset{x \to 1}{\text{Lt}} \frac{f(x) - f(1)}{x - 1} = f^1(x)$

$f^1(1) = 1\left(\frac{1}{1+1}\right) + \text{Tan}^{-1}(x)$

$= \frac{1}{2} + \frac{\pi}{4} = \frac{2+\pi}{4}$

15. (B) $xy = (x + y)^n$

$\Rightarrow \frac{dy}{dx} = \left(\frac{x+y-nx}{ny-x-y}\right)\frac{y}{x}$

but given $\frac{dy}{dx} = \frac{y}{x}$

$\therefore$ $\frac{x+y-nx}{ny-x-y} = 1 \Rightarrow n = 2$

16. (A) $B - A = B - (A \cap B)$

$P(B - A) = P(B) - P(A \cap B)$

$P(B) = P(B - A) + P(A \cap B)$

$= \frac{8}{25} + \frac{3}{25} = \frac{11}{25}$

17. (C) $n(S) = {}^6C_3 = 20$, $n(A) = 2$

[In the hexagon ABCDEF; ACE, BDF are equilateral]

$\Rightarrow P(A) = \frac{2}{20} = \frac{1}{10}$

18. (A) $f(y) = \frac{1-y}{1+y} = \frac{1 - \left(\frac{1-x}{1+x}\right)}{1 + \left(\frac{1-x}{1+x}\right)}$

$= \frac{\frac{(1-x)-(1-x)}{\cancel{(1+x)}}}{\frac{(1+x)+(1-x)}{\cancel{(1+x)}}} = \frac{\cancel{1} + x - \cancel{1} + x}{1 + \cancel{x} + 1 - \cancel{x}}$

$\Rightarrow \frac{\cancel{2}x}{\cancel{2}} = x$

19. (B) Let the numbers are a and b. Then, we have

$\frac{2ab}{a+b} = -\frac{8}{5}$ and $\sqrt{ab} = 2$

$\Rightarrow \frac{2 \times 4}{a+b} = -\frac{8}{5}$

$\Rightarrow a + b = -5$

Now, (2a) (2b) = 4ab =16

and 2a + 2b = 2 (a + b) =2 (–5) = –10

$\therefore$ Required quadratic equation is

$x^2 + 10x + 16 = 0$

20. (C) Given, ABCD is a parallelogram with vertices

A(4, 4, –1), B(5, 6, – 1), C(6, 5, 1) and D(x, y, z).

We know that diagonals of parallelogram ABCD bisects each other.

$\therefore$ Mid-point of AC = Mid-Point of BD

$\Rightarrow \left(\frac{4+6}{2}, \frac{4+5}{2}, \frac{-1+1}{2}\right) = \left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$

$\Rightarrow \left(\frac{10}{2}, \frac{9}{2}, 0\right) = \left(\frac{x+5}{2}, \frac{y+6}{2}, \frac{z-1}{2}\right)$

On comparing both sides, we get

$\frac{x+5}{2} = \frac{10}{2}, \frac{y+6}{2} = \frac{9}{2}$ and $\frac{z-1}{2} = 0$

$\Rightarrow x + 5 = 10, y + 6 = 9$ and $z - 1 = 0$

$\Rightarrow x = 10 - 5, y = 9 - 6$ and $z = 1$

$\Rightarrow x = 5, y = 3$ and $z = 1$

Thus, D(x, y, z) = D (5, 3, 1)

21. (D) Given, $f(x) = \sqrt{\log_{0.5} x!}$

$f(x)$ is defined when

$\log_{0.5} x! \geq 0$

$\Rightarrow x! \leq (0.5)^\circ$

$\Rightarrow x! \leq 1$

$\therefore x \in \{0, 1\}$

22. (D) We have x_1, x_2, x_3 and y_1, y_2, y_3 are in GP with the same common ratio.

Let r be the common ratio.

$\therefore x_1 = x, x_2 = xr$ and $x_3 = xr^2$

Similarly, $y_1 = y$

$y_2 = yr$ and $y_3 = yr^2$

$$\therefore \text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} x & y & 1 \\ xr & yr & 1 \\ xr^2 & yr^2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}xy\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = \frac{1}{2}\times 0 = 0$$

[$\because$ C_1, C_2 are identical]

$\therefore$ The given points are collinear.

23. (C) Given, $y = \log_2(\log_2 x)$

$$\Rightarrow \quad y = \log_2\left(\frac{\log x}{\log 2}\right) \quad \left[\because \log_a b = \frac{\log b}{\log a}\right]$$

$$\Rightarrow y = \frac{\log \frac{\log x}{\log 2}}{\log 2}$$

$$\Rightarrow y = \frac{\log(\log x) - \log(\log 2)}{\log 2}$$

$$\left[\because \log\frac{a}{b} = \log a - \log b\right]$$

$$\therefore \frac{dy}{dx} = \frac{1}{\log 2}\left[\frac{1}{\log_e x}\times\frac{1}{x} - 0\right] = \frac{1}{\log 2 . \log_e x . x}$$

$$= \frac{1}{(x\log_e x)\log_e 2}$$

24. (B) We have,

$$\overline{Z}^{\frac{1}{3}} = a + ib$$

$$\Rightarrow \overline{Z} = (a+ib)^3$$

$$\Rightarrow x - iy = (a+ib)^3 \qquad [\because \overline{z} = x - iy]$$

$$\Rightarrow x - iy = a^3 + i^3b^3 + 3a^2(ib) + 3a(i^2b^2)$$

$$\Rightarrow x - iy = a^3 - ib^3 + 3a^2bi - 3ab^2$$

$$\Rightarrow x - iy = (a^3 - 3ab^2) + i(3a^2b - b^3)$$

$$\Rightarrow x = a^3 - 3ab^2 \text{ and } y = -3a^2b + b^3$$

$$\Rightarrow \frac{x}{a} = a^2 - 3b^2 \text{and} \frac{y}{b} = -3a^2 + b^2$$

Now, $\frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 - 3a^2 + b^2$

$= -2a^2 - 2b^2$

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = -2(a^2 + b^2)$$

$$\therefore \frac{1}{a^2+b^2}\left(\frac{x}{a} + \frac{y}{b}\right) = -2$$

25. (C) We have,

$$|x|^2 - 5|x| + 6 = 0$$

Let $|x| = y$

$\Rightarrow y^2 - 5y + 6 = 0$

$\Rightarrow (y-2)(y-3) = 0$

$\Rightarrow y = 2, 3$

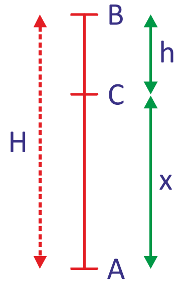
$\Rightarrow |x| = 2$ or $|x| = 3$

$\Rightarrow x = \pm 2$ or ± 3

$\therefore$ Number of real roots are 4.

PHYSICS

26. (B) Let T be the time of ascent and H be the total height. Then T = u/g



And $H = uT - \frac{1}{2}gT^2$

Let (T – t) be the time taken by the ball to go from A to C. The distance covered in time (T–t) is

$x = u\,(T-t) - \frac{1}{2}g\,(T-t)^2$

So, distance covered by ball in last t seconds.

$$h = H - x = \left[uT - \frac{1}{2}gT^2\right]$$

$$-\left[u(T-t) - \frac{1}{2}g(T-t)^2\right]$$

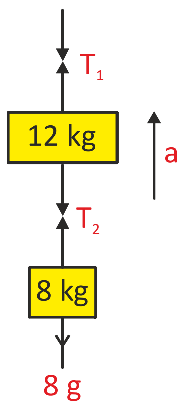
$$= ut - gt\,T + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \;[\because T = u/g]$$

27. (B) It is clear from the figure given below, the equation of motion of 8 kg block is

$8 \times a = T_2 - 8\,g$

$T_2 = 8a + 8g = 8\,(a + g)$

$= 8 \times (2.2 + 9.8) = 96$ N



The equation of motion of 12 kg block is

$12 \times a = T_1 - 12\,g - T_2$

$T_1 = 12(a + g) + T_2$

$= 12\,(2.2 + 9.8) + 96 = 240$ N.

28. (D) We know, elongation in wire $(\Delta L) = \frac{FL}{AY}$

or $F = \frac{AY\Delta L}{L}$ ($\because$ F = normal force)

Given:

Material of both wires is same $\therefore Y_A = Y_B$

Elongation in both wires A and B are equal

$\therefore \Delta L_A = \Delta L_B$

So, $F \propto \frac{A}{L}$

$$\frac{F_A}{F_B} = \frac{A_A}{L_A} \times \frac{L_B}{A_B} \quad \text{But} \quad \frac{A_A}{A_B} = \frac{\pi r_A^2}{\pi r_B^2} = \frac{r_A^2}{r_B}$$

$$\frac{F_A}{F_B} = \frac{r_A^2}{r_B^2} \times \frac{L_B}{L_B} = (2)^2 \times \frac{1}{4} = 1$$

$$\left[\because \frac{r_A}{r_B} = \frac{2}{1} \text{ and } \frac{L_A}{L_B} = \frac{4}{1}(\text{given})\right]$$

29. (A) $K = \frac{r_1^2 + r_2^2 + \ldots\ldots}{n}$, radius of gyration depends on the distribution of mass about the axis of rotation and it is independent of mass of the body.

30. (D) Arial Magnification $= \frac{\text{Area of image}}{\text{Area of object}}$

$= 1.55 / 1.75 \times 10^4 = 8857$

Linear Magnification $= \sqrt{8857} = 94.11$

31. (A) The resultant of three vectors cannot be zero if one vector does not lie in between the sum and difference value of the two other vectors.

One force must lie in between the sum and difference of two other forces.

32. (C) Let x be the distance of point from the moon where, the gravitational field intensity is zero. The distance of point from the earth = $(60R - x)$.

So, $\frac{G(M/81)}{x^2} = \frac{GM}{(60R - x)^2}$

or $\frac{1}{9x} = \frac{1}{60R - x}$

or 60 R = 10 x or x = 6 R

33. (D) Here, R = 2.8 / 2 = 1.4 mm = 0.14 cm;

$\frac{4}{3}\pi R^3 = 125 \times \frac{4}{3}\pi r^3$

or $r^2 = R/5 = 0.14/5 = 0.028$ cm.

Change in energy = S.T. × increase in area

$= 75 \times [125 \times 4\pi r^2 - 4\pi R^2]$

$= 75 \times 4\pi \times [125 \times (0.028)^2 - (0.14)^2]$

= 74 erg

34. (B) As water enters into the vessel A, it becomes heavier. Gravity helps it to sink. External work required for immersing A is obviously less than that for immersing B.

35. (D) When difference in temps. of a liquid and the surroundings is small ($\approx$ 30°C), then

$-\frac{dQ}{dt} \alpha (\theta - \theta_0)$

For numerical problems, when a body cools from θ_1 to θ_2 in time t, then

$\frac{\theta_1 - \theta_2}{t} = \alpha\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$

According to Newton's law of cooling, rate of cooling $\propto$ temp. diff. between the liquid and surroundings. As temp. diff. decreases gradually, time taken to cool increases i.e. $T_3 > T_2 > T_1$ or $T_1 < T_2 < T_3$

36. (D) Given, $x = 0.20$ m; $y = 0.20$ m, u = 1.8 m/s.

Let the ball strike the nth step of stairs.

Vertical distance travelled

$= ny = n \times 0.20 = \frac{1}{2}gt^2$

Horizontal distance travelled = n x = ut

or t = nx/u

$\therefore \quad ny = \frac{1}{2}g \times \frac{n^2 x^2}{u^2}$

or

$n = \frac{2u^2}{g}\frac{y}{x^2} = \frac{2 \times (1.8)^2 \times 0.20}{9.8 \times (0.20)^2} = 3.3 \approx 4$

37. (B) $\upsilon_e = \sqrt{2GM/R}$ i.e. $\upsilon_e \, \alpha \, 1/\sqrt{R}$

$\therefore \quad \frac{\upsilon_{e_1}}{\upsilon_{e_2}} = \sqrt{\frac{R_2}{R_1}}$ or $\frac{1}{100} = \frac{R_2}{R_1}$

or $R_2 = \frac{R_1}{100} = \frac{6400}{100} = 64$ km

38. (D) Under isothermal conditions, T = constant.

$\therefore$ Internal energy = constant i.e. change in internal energy is zero.

39. (C) Energy does not have the units of kg-m/sec.

Unit of energy is joule.

40. (A) $\upsilon = 1.5$ m/s, $\frac{dm}{dt} = 5$ kg/s

$F = \frac{dm}{dt} \times \upsilon = 5 \times 1.5 = 7.5$ N

$P = F \times \upsilon = 7.5 \times 1.5 = 11.25$ W

CHEMISTRY

41. (A) The number of electrons in Na^+ = 11 – 1 = 10

The number of electrons in Ne = 10

The number of electrons in K^+ = 19 – 1 = 18

The number of electrons in O = 8

Thus, Na^+ and Ne are isoelectronic with one another.

42. (B) P_1 = 1.00 atm P_2 = 0.80 atm

V_1 = 175 L V_2 = ?

As temperature remains constant, hence $P_1 V_1 = P_2 V_2$ (Boyle's law)

$$V_2 = \frac{P_1V_1}{P_2} = \frac{1\,\text{atm} \times 175\,\text{L}}{0.80\,\text{atm}} = 218.75\text{ L}$$

43. (D) For coordinate bond formation, there should be a lone pair of electrons which the H_2 molecule does not have.

44. (C) KI_3 and $CuSO_4$ give 2 ions whereas K_2HgI_4 gives 3 ions. $FeCl_3$ gives 4 ions.

45. (C) Mass of $NaNO_3$ = 0.38 g

Volume of the solution = 50.0 mL

Molar mass of $NaNO_3$ = 23 g/mol + 14 g/mol + 3 × 16 g/mol

= (23 + 14 + 48) g/mol = 85 g/mol

Amount of $NaNO_3$ dissolved

$$= \frac{0.38\,\text{g}}{85\,\text{g/mol}} = 4.47 \times 10^{-3}\text{ mol}$$

Molarity of the solution

$$= \frac{4.47 \times 10^{-3}\,\text{mol}}{50.0\,\text{mL}} \times 1000\text{ mL/L}$$

$= 0.089\text{ mol L}^{-1}$

46. (A) B.O. in N_2 = (10 – 4)/2 = 3

B.O. in O_2^{2+} = (10 – 4)/2 = 3

B.O. in O_2^- = (10 – 5)/2 = 2.5

B.O. in N_2^- = (10 – 5)/2 = 2.5

B.O. in O_2 = (10 – 6)/2 = 2

B.O. in O_2^+ = (10 – 5)/2 = 2.5

Thus, N_2 and O_2^{2+} have identical bond order of 3.0

47. (D)

(a) O.N. of Cl^- = – 1

(b) O.N. of Cl in ClO^- = $x - 2 = -1$ or $x = +1$

(c) O.N. of Cl in ClO_2^- = $x + 2 \times (-2) = -1$ or $x = +3$

(d) O.N. of Cl in ClO_3^- = $x + 3 \times (-2) = -1$ or $x = +5$

48. (D) One electron in the outermost shell of the given group 1 elements causes them to have similar properties.

49. (B) Ethyl alcohol undergoes combustion according to the reaction,

$$C_2H_5OH + 3O_2 \rightarrow 2CO_2 + 3H_2O \quad \Delta H = -1367\text{ kJ mol}^{-1}$$

Then $\Delta_c H = \sum aH_{products} - \sum bH_{reactants}$

Since, the enthalpy of a compound is taken as equal to its heat of formation, and the enthalpy of an element is taken as zero, we can write,

$$-1367 = [2\Delta_f H\,(CO_2) + 3\Delta_f H(H_2O)] - [\Delta_f H(C_2H_5OH) + 0]$$

Therefore, $\Delta_f H\,(C_2H_5OH)$ = 2 (– 393.4) + 3 (– 285.9) + 1367 = – 277.5 kJ mol^{-1}

50. (B) Element Y belongs to group 14 of the periodic table which forms two chlorides YCl_4 (a colourless, volatile liquid) and YCl_2 (a colourless solid).

51. (B) Reaction is reversed. K = 1/0.6 = 1.67.

52. (B) CH_2N_2 is called diazomethane (diazo + methane).

53. (B) Electronic configuration of Z = 105,

$n + l = 8$, for 5f = (5 + 3) = 8 and for

6d = (6 + 2) = 8 and electrons present in 5f = 14

and electrons present in 6d = 3,

Thus, total no. of electrons = 14 + 3 = 17

54. (C) Gases do not have any definite volume. Liquids have definite volume.

55. (A) $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O,\ \Delta H_1 = -x$ kJ

$$CH_3OH + \frac{3}{2}O_2 \rightarrow CO_2 + 2H_2O, \Delta H_2 = -y \text{ kJ}$$

Subtracting (ii) from (i), we get

$$CH_4 + \frac{1}{2}O_2 \rightarrow CH_3OH, \Delta H_3 = -\text{ve}$$

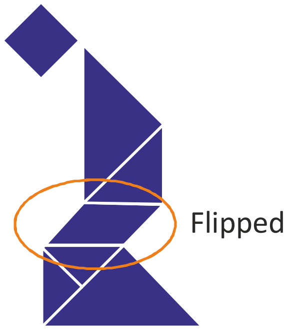
i.e., $-x - (-y) = -$ ve

$y - x = -$ ve

Hence, $x > y$.

CRITICAL THINKING

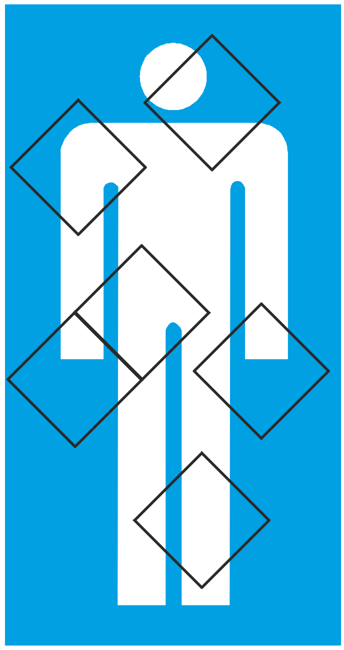
56. (D)



57. (A) Data in Statement I alone is sufficient to answer the question, while the data in Statement II alone is not sufficient to answer the question.

58. (D) If both I and II are implicit

59. (B)



60. (C) Since the weight is 10 Kg and there are 4 sections of rope supporting it, then by dividing 10 by 4, you will get 2.5 Kg. In all cases, just divide the weight by the number of sections of rope supporting it to get the force needed to lift the weight.